The development of a mathematical model for the prediction of the residence time distribution of a hydrodynamic vortex separator

Le développement d’un modèle mathématique pour la prévision des temps de séjour dans un régulateur hydrodynamique à vortex

Dr. R.M. Alkhaddar[1] , Miss C.H. Cheong[2] , Dr. D.A. Phipps[3] , Professor R. Andoh[4] , Dr. A. James[5] , Dr. Philip Higgins[6]

[1] Reader in Water Engineering, Liverpool John Moores University, Clarence Street, Liverpool L3 5UG, UK
[2] MPhil Student, Liverpool John Moores University
[3] Reader in Applied Chemistry, Liverpool John Moores University
[4] Visiting Professor, LJMU and Technical Director, Hydro International Plc, Shearwater House, Victoria Road, Clevedon BS21 7RD, UK
[5] Lecturer, Department of Chemical Engineering, UMIST, PO Box 88, Manchester M60 1QD, UK
[6] Research Fellow, School of the Environment, Coventry University

ABSTRACT

This study considers a mathematical model which describes macromixing patterns within a Hydrodynamic Vortex Separator (HDVS). The basic model has a combination of 3 Continuous Stirred Tank Reactors (CSTRs) in series, together with a slow-mixing zone and a by-passing flow element. It is primarily an extension of the Tank-In-Series Model (TISM). Using a series of different boundary conditions it was used to simulate the Residence Time Distribution (RTD) characteristics of an HDVS which were compared with the observed characteristics derived from lithium chloride (LiCl) tracer studies. The mathematical model was successfully tested with results from a laboratory scale unit (60*l* in volume) and a prototype (464*l* in volume) both operated with no baseflow component. A scale factor between the model and prototype has been derived. The scale factor was found to have a value greater than that given by the Froude and the Hazen models. This will mean a more economical design of these units and a more efficient performance.

RESUME

Cette communication étudie un modèle mathématique reproduisant le macro-mélange dans un séparateur hydrodynamique à vortex *(HDVS).* Le modèle de base combine trois réacteurs réservoirs à mouvement continu *(CSTRs)* en série, ainsi qu’une zone de mixage lent et un élément de dérivation du flot. C’est essentiellement un développement du modèle de réservoirs en série *(TISM).* Prenant en compte différentes séries de conditions limites, il a été utilisé pour simuler les caractéristiques des temps de séjour *(RTD)* d’un *HDVS*. Elles ont été comparées avec les caractéristiques observées lors d’études de traçage faites avec le Lithium de Chlore *(LiCl)*. Le modèle mathématique a été testé avec succès avec les résultats d’un modèle de laboratoire (volume de 60*l*) et d’un prototype (volume de 464*l*), les deux fonctionnant sans débit de base*.* Un coefficient d’échelle entre le modèle et le prototype a été dégagé. Le coefficient d’échelle s’est avéré avoir une valeur supérieure à celle obtenue avec les modèles de Froude et de Hazen. Cela signifie une conception plus économique de ces unités ainsi que de meilleures performances.

KEYWORDS

Combined Mathematical (CM) model; Hydrodynamic Vortex Separator (HDVS); Residence Time Distribution (RTD); Scale factor

1. INTRODUCTION

The Hydrodynamic Vortex Separator (HDVS) operates in a combined vortex manner and is currently used in the fields of urban stormwater management and wastewater treatment.

The specially designed internal components for a HDVS typically consist of a baffle plate, dip plate, a centre cone and an effluent spillway. A sludge hopper on the bottom tank is designed to collect sludge or solids when it operates for solid-separation. Removal is through a baseflow component at the lowest part, as shown in Figure 1. The flow regime in the device is a complex turbulent multi-phase flow with turbulence and shearing effects. The HDVS history, operation, design and performance in various application have been documented (*Hedges* 1991; *Fagan* 1993; *Andoh et al.* 1993, 1994, 1998, Brombach, 1993).

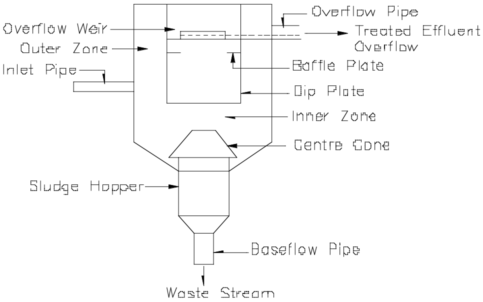


Figure 1: Typical Hydrodynamic Separator Configuration.

As well as being a separator the HDVS can be used as a contact tank for disinfection and for chemically assisted solid removal. Previous work investigated the potential use of a HDVS as a mixing device, with particular consideration of the chemical processes used for tertiary treatment. This was done with different flow configurations and with two different sizes of separators (*Andoh et al.* 1993, 1994,1998).

If the mixing behaviour of the system is understood, then the efficiency of disinfection or coagulation could be better predicted. For example, under conditions of excess disinfectant, the effective kill can be predicted if the RTD and the pseudo first-order rate constant (K_1) for bacterial death are known *(Alkhaddar et al.* 2000).

2. STUDY OBJECTIVES

The aim of this study is to investigate the flow characteristics of a small scale HDVS with an appropriate range of total flow rate but with no baseflow. Previous studies *Alkhaddar et al.* (1999) and *Higgins et al.* (1998, 1999 (a) & (b)) introduced two single-parameter mathematical models to describe the flow patterns and to predict conversion for a HDVS. However, these models are not adequate to identify and quantify any stagnancy and short-circuiting in the HDVS. Hence, the first aim was to construct a more complex mathematical model in order to evaluate their significance.

Meanwhile, much work has been reported on physical and chemical tests on different types of HDVSs (*Dudley et al.* 1993*, Alkhaddar et al.* 1999, *and Higgins et al.* 1999 (b)). In particular, the residence time distributions of different types of overflow structures have been compared. Less attention has been given to the scaling laws, which relate the results from such model tests to real installations. Therefore, the second aim of this study is to investigate the scaling of HDVSs with respect to the Residence Time Distribution (RTD). This can be developed further to predict the RTD performance of a particular sized device.

3. RESIDENCE TIME DISTRIBUTION (RTD)

In any reactor, the distribution of residence time can significantly affect its performance. By definition for a tubular reactor, under a plug-flow condition, the residence-time is the same for all elements of the effluent fluid. However, for a continuous stirred tank reactor (CSTR), there is a spread of residence times, where different elements of fluid passing through the system will stay for different times (*Levenspiel* 1972). For a HDVS, the RTD and mixing regime is neither plug flow nor perfect mixing. It is because the velocity distribution within the HDVS is generally not uniform resulting in different residence times of fluid elements travelling along different streamlines. This distribution of residence time can have a significant effect on the device's performance for various applications. The appropriate function of RTD can be represented by *exit age distribution, E(t)* (*Levenspiel* 1972).

4. MATERIALS AND METHODS

The small-scale HDVS investigated was a 375mm diameter, constructed of transparent perspex. Volume was approximately 60 litres (*l*), excluding any connecting pipework. This apparatus was half-scale of a prototype HDVS. The prototype device was a mild-steel free-standing HDVS, diameter 750mm with a volume of 464 *l*. The flows entered the device through a 40-mm diameter horizontal pipe. An injection point was located at approximately 12-pipe diameters from the device. This was used for tracer tests and dye (washout) tests. A calibrated rotameter was placed at the inlet pipe for measurement of inlet flow, which was controlled by a gate valve. All experiments were conducted with no baseflow, for a range of influent flow rates, i.e. 6 – 90 *l*/min and 30 - 480 *l*/min, respectively.

In this study, a pulse injection technique was used to obtain the RTD. This was obtained experimentally by injecting a tracer, lithium chloride (LiCl), in solution into the device's inlet pipe (pipe centre) at $t = 0$ and then measuring the outlet concentration in the outlet, as a function of time. Samples were analysed immediately after collection using an atomic absorption spectrophotometer (Perkin-Elmer 372). Dye (wash-out) tests were conducted to determine the model parameter for slow-mixing volume and to observe the flow regime within HDVS. Dylon ™ dye was used as the tracer for the dye (washout) tests.

5. DEVELOPMENT OF THE COMBINED MATHEMATICAL (CM) MODEL

The mathematical analysis of HDVS operation has generally been approached through the Axial Dispersion model (ADM) or the Tank-In-Series model (TISM). However, in some cases of mixing within the HDVS, the shape of RTD curves cannot be approximated accurately either by the TISM or ADM. Although these two models can be used to describe some non-ideal flow behaviour they are inadequate for a correct description of "tailing" on RTD curves and errors in the mass balance. Therefore, a combined mathematical (CM) model has been developed which can be used to determine the influence of any by-passing or of any slow-mixing regions within the device. This will provide more flexibility in fitting to the experimental RTD curves compared to previous one-parameter models. The model presented here only applies to the HDVS operated without a baseflow component.

The CM model is an extension of the TISM model and is intended to be a simple representation of physical reality. It is believed that a real HDVS can be described by a combination of three continuous stirred tank reactors (CSTR) one of which can exchange with a slow-mixing zone. In addition the inlet and outlet can be interconnected with a by-pass which has an adjustable flow rate which can be set to zero. Tanks 1, 2 & 3 are assumed to have ideal CSTR behaviour and are connected in series, having volumes of V_1, V_2 & V_3 respectively. Tank 4 with a volume of V_4 is assumed to be a slowly exchanging region. It is suggested that this region is subjected to relatively slow-mixing rather than being completely stagnant. In the other words, the tanks 1, 2 & 3 are considered as 'active' volume, and the liquid in these tanks undergoes ideal mixing.

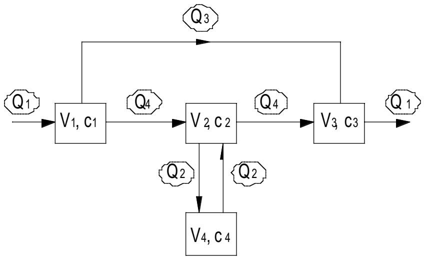


Figure 2. Configuration of Combined Mathematical (CM) model without baseflow
Where, Q_1*= Influent flow rate (l/min.);* Q_2*= Exchange flow rate (l/min.)*
Q_3 *= By-pass flow rate (l/min.),* Q_4 *= Active or effective mixing flow rate (l/min.)*
C_3 *= Effluent tracer concentration(mg/l);* C_1*,*C_2 *= Cons. in tanks 1&2 (mg/l)*
V_1*,* V_2*,* V_3 *= Active or effective mixing volumes (l);* V_4 *= Slow-mixing volume (l)*

Ideally Q_1 is divided between two paths, which are Q_3 and Q_4, hence, $Q_1 = Q_3 + Q_4$

If there are no losses within the system, then the effluent flow rate is equal to the influent flow rate (Q_1). An exchange flow rate of Q_2 exits between the slow-mixing zone and the ideally mixed zone i.e. between tank 2 and tank 4.

6. ANALYTICAL SOLUTIONS

By considering the HDVS as a closed-closed system with incompressible fluids, the inlet flow is always the same as the outlet flow, although the flow varies with time. Therefore, the volume of the system is constant. The model is based on the theoretical tracer mass balance in each reactor, i.e. **Accumulation = in - out.** The material recovery must be 100 % if there is no material left in the system after a certain time. The tracer total amount (M) can be calculated from the following.

$$M = \int_0^{\infty} C Q_0 \, dt$$

For the first CSTR (tank 1), because the reactor is assumed to be perfectly mixed, the concentration of the tracer either in the effluent or within the reactor integrating with $C=C_0$ at $t=0$ yields (where C_0 is the initial concentration of tracer), $C_1 = C_0 e^{-k_1 t}$

The initial mass balances of each reactor are:

Reactor1: $V_1 \dfrac{dC_1}{dt} = -Q_3 C_1 - Q_4 C_1$; Reactor 2: $V_2 \dfrac{dC_2}{dt} = Q_4 C_1 - Q_4 C_2 + Q_2 C_4 - Q_2 C_2$

Reactor 3: $V_3 \dfrac{dC_3}{dt} = Q_4 C_2 - Q_1 C_3 + Q_3 C_1$; Reactor 4: $V_4 \dfrac{dC_4}{dt} = -Q_2 C_4 + Q_2 C_2$

Let $k_{31} = \dfrac{Q_3}{V_1}$, $k_{41} = \dfrac{Q_4}{V_1}$, $k_{42} = \dfrac{Q_4}{V_2}$, $k_{22} = \dfrac{Q_2}{V_2}$, $k_{43} = \dfrac{Q_4}{V_3}$, $k_{13} = \dfrac{Q_1}{V_3}$, $k_{33} = \dfrac{Q_3}{V_3}$, $k_{24} = \dfrac{Q_2}{V_4}$

And let $\tau = (k_{42} + k_{22} + k_{24})$, $\rho = k_{42} k_{24}$, $\varepsilon = (k_{24} - k_{31} - k_{41}) k_{42} C_0$

The effluent concentration (C_3) of the HDVS is represented by a second-order differential equation. The complete solution of such a non-homogenous differential equation is well known as it can be derived in the normal way from the sum of the complementary function (C.F.) and particular integral (P.I.), as derived by *Wetner et al*. (1986). Therefore, the effluent concentration of a system,

$$\boldsymbol{C_3 = A_1 e^{-k_1 t} + B_1 e^{m_1 t} + D_1 e^{m_2 t} + E_1 e^{-k_{13} t}} \qquad [1]$$

where $A_1 = \dfrac{(k_{43} M_0 + k_{33} C_0)}{(k_{13} - k_1)}$, $B_1 = \dfrac{k_{43} A}{(m_1 + k_{13})}$, $D_1 = \dfrac{k_{43} B}{(m_2 + k_{13})}$

and, $A = \dfrac{m_1 m_2 (M_0 - C_0) + k_1 M_0 m_1}{k_1 (m_2 - m_1)}$, $B = \dfrac{m_1 m_2 (M_0 - C_0) + k_1 M_0 m_2}{k_1 (m_1 - m_2)}$

$$E_1 = -(A_1 + B_1 + D_1), \; M_0 = \frac{\varepsilon}{k_1^2 - \tau k_1 + \rho}$$

$$k_1 = k_{31} + k_{41}\ ,\ m_1 = \frac{-\tau}{2}[1 - \sqrt{(1 - \frac{4\rho}{\tau^2})}]\ ,\ m_2 = \frac{-\tau}{2}[1 + \sqrt{(1 - \frac{4\rho}{\tau^2})}]$$

7. ASSUMPTIONS

The use of the mathematical model under investigation required a choice of both boundary conditions and model parameters. No unique set of assumptions could be made and the selection was empirical. However, it was essential to minimise the number of the parameters to two in each case as more than 2 adjustable parameters may reduce the power of any model by allowing a fit to almost any data without discrimination. Three models 1, 2 and 3 were established with different considerations. Each is limited to having two adjustable parameters, the slow-mixing volume (V_4), the exchange flow rate (Q_2) or the by-pass flow rate (Q_3), see Table 1.

Table 1: The three proposed Combined Mathematical (CM) models

Model No.	Boundary condition	Model parameters
Model 1	$V_1 = V_2 + V_4 = V_3$ & $V_4 = V_{total}/3 - V_2$	V_4 & Q_2 Where $Q_3 = 0$
Model 2	$V_1 = V_2 = V_3$ & $V_4 = V_{total} - 3V_1$	V_4 & Q_2 Where $Q_3 = 0$
Model 3	$V_1 = V_2 = V_3$ & $V_4 = V_{total} - 3V_1$	Q_3 & Q_2 Where $Q_3 \neq 0$

8. DATA ANALYSIS AND RESULTS

There were Two sets of experimental data available for testing the mathematical model. They were derived from the 375mm-diameter and the 750mm-diameter devices respectively. Several methods have been examined to test the correlation between the experimental and the mathematical model data, but the Chi-Squared (χ^2) method produced the most consistent results.

The volume (V_4) in model 1, limited to one third of the total volume seemed to have a poor goodness of fit. After releasing the limitation on V_4, Model 2 produced a better fit. However, the estimated V_4 was not in agreement with the evidence from the dye wash-out tests. The predicted value for V_4 (from CM model) was always higher than the measured slow-mixing volume at the sludge hopper.

Models 1&2 were similar in that the adjustable parameters were always set as (V_4) and (Q_2), differences were then restricted to the boundary conditions. When Model 3 was developed, the parameter V_4 was replaced by the experimental value of slow-mixing volume in the sludge hopper, determined from the dye (wash-out) tests. The increase in the goodness of fit coincided with an inclusion of a non-zero by-pass in the CM model. Therefore, the adjustable parameters of Model 3 were (Q_3) and (Q_2).

It can be concluded that V_4 as described in the CM model should be considered as a non-effective volume which tends to reduce the mixing volume of the HDVS. This volume is not simply the slow-exchange component, but could also be caused by the phenomenon of by-passing. The results of Model 2 provided the magnitude of non-effective volumes ranging from 23 *l* –15 *l* (38%-24%) at flow rates of 6 *l*/min.- 90 *l*/min. and 174 *l*– 76 *l* (37%-16%) at flow rates of 30 *l*/min.- 480 *l*/min. for the 375mm-diameter and the 750mm-diameter devices respectively.

The experimental evidence showed that by-passing increased with the influent flow rate. The phenomenon of by-passing will reduce the mean residence time of the tracers in the HDVS, therefore, the peak of the curve would occur at an earlier time. This is in contrast to the effect of flow on the slow-mixing volume. As the flow increases, the slow-mixing volume decreases which would tend to increase the mean residence time. Hence, the optimum design flow rate that balances these two effects to give the minimum non-effective volume can be approximately evaluated. The estimated optimum design flow rates for the 375-mm diameter and the 750-mm diameter devices ranged from 8 –15 *l*/min and 90-120 *l*/min respectively. Within the range of the practical flow rates, the CM models give a good fit to the experimental RTD data, particularly using Model 3, as shown in Figure 3.

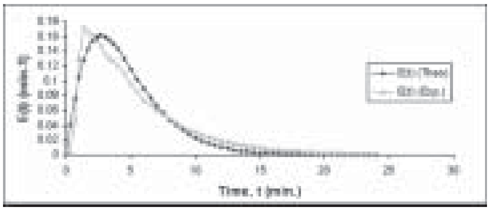
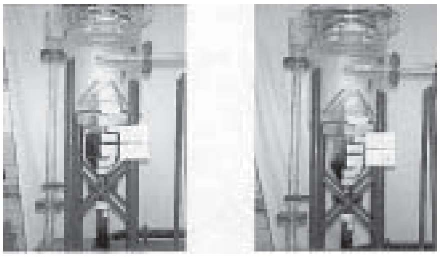


Figure 3: Comparison of Theoretical (Theo.) and Experimental (Exp.) E(t) *data at selected influent flow rate of 15 l/min. for a 375mm-diameter device without baseflow.*

Observation of dye (wash-out) test

It was observed that the HDVS tends to have a significant volume fraction which is best described as ‘slow-mixing’. This fraction is greatest at low flow rates and decreases as the flow rate increases. Figure 4 clearly identifies that the HDVS was divided into two main parts. The upper part which was clear and moving as fast as the incoming flow and the lower part, largely contiguous with the sludge hopper which was a very slow-moving zone. At low flow rates, the fluid remained much longer in the system. At high flow rates, the dyed part was washed away very quickly and it completely disappeared when the flow rate was increased to 22 l/min.

The tests also showed that the sludge hopper should not be removed from the active volume particularly at high flows. At the high flow rates, the slow-mixing zone does not occupy the whole sludge hopper, but, it is mainly at the centre cylinder region. Since part of the sludge hopper still has an ideal mixing, it is not suggested to remove the sludge hopper when considering the ‘active volume ‘ of the device.



(a) At flow rate of 8 l/min *(b) At flow rate of 15 l/min.*

Figure 4: Photographs illustrating the phenomenon of slow-mixing volume in the 375mm-dia device without baseflow at selected flow rates.

The Development of a scaling relationship with respect to RTD

Currently used scaling laws for the HDVS.

The scale-up of reactors such as HDVS for solids removal efficiency has been considered previously, *Tyack et al.* (1997), but scale-up as a chemical reactor is not as straightforward since mixing significantly affects conversion. Yields can only be successfully predicted through the use of models which account for all the important effects. Since the hydrodynamic phenomena are themselves non-linear and, in turn, interact with mixing and reaction phenomena in a highly non-linear way, only models that include the relevant relationships will prove generally adequate. The currently used scaling laws for solids removal efficiency giving scale factors of 2.5 and 2.0 are:

Froude's law, $\frac{Q_p}{Q_m} = (\frac{L_p}{L_m})^{2,5}$; Hazen's law, $\frac{Q_p}{Q_m} = (\frac{L_p}{L_m})^{2}$

Where Q_p, Q_m = Flow rate in the prototype and the model respectively
L_p, L_m = A characteristic length in the prototype and the model respectively

Scaling as a reactor

The scaling relationship presented in this work was aimed at predicting the RTD performance of full-scale HDVS units from the behaviour of geometrically similar devices of smaller scale. To achieve the similar material recoveries on both devices the shape of the RTD curve produced by the small and large-scale HDVS units must be similar. The shape of an RTD curve is dependent on the mean residence time (t_m) and the variance (σ^2) of the distribution, which can be determined by the method of moments. To produce the same RTD curves on the small and large scale devices, it is necessary to obtain simultaneous similarity of both the mean residence time (i.e. the centroid of the curve) and the variance (i.e. the width of the curve). Moreover, both of these HDVS devices would then have the same mixing limit.

When HDVSs are used as a contactor for disinfection, it is important to maintain the residence time constant irrespective of size. Scaling can be done via dimensional analysis and if the ratio of the residence times at both sizes is set to 1 then it follows that $Q_r = L_r^3$. Dimensional analysis then gives a good fit for the mean residence time but it does not produce a satisfactory similarity of variances. Conversely, the Froude scaling law ($Q_r = L_r^{2.5}$) produced a better fit for the similarity of variances compared to dimensional analysis. However, scaling-up by the Froude number did not provide the similarity of mean residence times. In order to achieve the balance between the similarity of mean residence times and of variances a scale factor of 2.85 was adopted. Therefore, the new scaling equation can be derived as:

$$\frac{Q_p}{Q_m} = \left(\frac{D_p}{D_m}\right)^{2.85} \qquad [2]$$

Compared to using the Froude number, *Tyack et al.* (1997), this relationship is simpler and allows the ratio of mean residence times to be maintained at approximately 1 during scale-up. By using the Froude number or similar law, it is not possible to obtain a similar RTD curve between small and large scaled systems, as the ratio of mean residence time in model and prototype cannot be approximated to 1.

9. ANALYSIS OF RESULTS

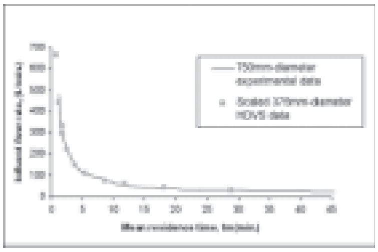


Figure 5: Results of scaling the RTD data using the new scaling relationship.

All experiments were conducted at the same conditions (temperature, viscosity of fluid and characteristics of tracer on both the model and prototype HDVS devices). RTD data was analysed by an empirical method and the new scaling relationship for the HDVS with respect to RTD was determined. The proposed scaling equation provided a good fit to the experimental data, as illustrated in Figure 5.

10. CONCLUSIONS

This study has successfully developed the RTD combined mathematical (CM) model to describe the mixing regime within the HDVS. The CM model provides insight into the physical reality of flow within the HDVS. Stagnancy, flow distribution, interchanges, by-passing, and the dimensionless variance (dispersion) of the residence time distribution all can affect the description of the imperfect plug-flow mixing regime and the non-ideal flow behaviour within the HDVS. Modelling and measurement of the residence time distribution (RTD) should be important aspects of design, analysis, and scale-up for a HDVS. There is a scarcity of good residence time data in this field of research, particularly data from experiments in which inlet and outlet conditions are correctly and rigorously monitored.

The scaling relationship proposed in this study between the investigated devices, which are the 375mm-diameter (model) HDVS and the 750mm-diameter (prototype) HDVS, has been evaluated based on an empirical method. In order to obtain the similarity of RTD on different sizes of HDVS, a new scaling relationship has been proposed to provide a reasonable compromise for the fit of RTD parameters on both devices. The proposed scaling equation i.e. $Q_p/Q_m = (D_p/D_m)^{2.85}$ showed that the goodness of fit to their experimental RTD curves was good.

However, Further details of this study should refer to *Cheong* 2000 and *Higgins* 2000. More experiments are being carried out in order to validate the mathematical model.

11. ACKNOWLEDGEMENTS

The authors wish to thank Hydro Int. Plc. for providing technical support and funding.

11. REFERENCES

Alkhaddar R.M., Higgins P.R. and Phipps D.A. (2000). Characterisation for In Situ Disinfection for a Hydrodynamic Vortex Separator (HDVS). *Proceedings of the 1st World Congress of the International Water Association, Paris, 3-7 July.*

Alkhaddar R.M., Higgins P.R., Phipps D.A. and Andoh R.Y.G., (1999). Residence Time Distribution of a Hydrodynamic Separator. *The CIWEM Journal ,* **13** (4), 255-261.

Andoh R.Y.G. (1998). Improving Environmental Quality using Hydrodynamic Vortex Separators. *Water Quality International, January/February. 47-51.*

Andoh R.Y.G., Harper I.(1993). The Swirl-Flo™ Process for High Rate Physio-Chemical Treatment at Wastewater Treatment Works. *Wat. Env. Federation–A Global Perspective for Reducing CSOs: Balancing Technologies, Costs and Water Quality, Louisville, Kentucky, US.*

Andoh R.Y.G., Smisson R.P.M. (1994). High Rate Sedimentation in Hydrodynamic Separators. *Proceedings of the 2nd International Conference On Hydraulic Modelling development and Application of Physical and Mathematical Models, Stratford, UK,* 341-358.

Brombach, H., Xanthopoulos, C., Hahn, H.H. and Pisano, W.C. (1993). Experience with Vortex Separators for Combined Sewer Overflow Control. *Wat Sci and Tech,Vol27, No.5-6, pp.93-104.*

Cheong C.H. (2000). MPhil Thesis. *Liverpool John Moores University*

Dudley J. and Mark H. (1993) Evaluation of the Swirl-Flow™ Process, *WRc Report UC 1869.*

Fagan G.W. (1993). A Quality Assessment of Dynamic Experiences: The UK Experience. *Proc. of the 6th Int. Conf on. On Urban Storm Drainage, Niagara Falls, Canada, 1663-1669*

Hedges P.D. (1991). The Performance of the Hydrodynamic Separator Combined Sewer Overflow. Practice and Performance. *Hydro Research and Development Ltd.*

Higgins P.R. (2000) PhD Thesis. *Liverpool John Moores University.*

Higgins P.R., Alkhaddar R.M. and Phipps D.A., (1999 (a)). The use of hydrodynamic vortex separators (HDVS) for distributed treatment of combined sewer overflows (CSO). *Hydro International Plc Conference on Sewer Networks and CSO Solutions for AMP3, Leicester,UK.*

Higgins P.R., Alkhaddar R.M. and Phipps D.A. (1999 (b)). The residence time distribution of a prototype hydrodynamic vortex separator operating with a baseflow component.*Proceedings of the 8th International Conference on Urban Storm Drainage, Sydney, Australia, Vol. 18-25.*

Higgins P.R., Alkhaddar R.M., Phipps D.A. and Andoh R.Y.G. (1998). Residence Time Distribution of a Hydrodynamic Separator. *Proceedings of the 2nd International Conference on Advanced Wastewater Treatment, Recycling and Reuse, Milan, Italy.*

Levenspiel O. (1972). Chemical Reaction Engineering. 2nd Edition. *John Wiley & Son.*

Tyack J.N., Fenner R.A. (1997). The use of scaling laws in characterising residence time in hydrodynamic separators. *Proceedings of the 2nd Int. Conf. On the Sewer and a Physical, Chemical and Biological Reactor, Aalborg, Denmark, pp 25-28 May.*

Weltner K., Grosjean J., Schuster P, and Weber W.J. (1986) Mathematics for engineers and scientists. *Stanley Thornes Ltd.*